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The Relation Between Rosenfeld' Fuzzy Subhyperring(hyperideal) and Fuzzy H_v-(Bi-ideal) in fuzzy.H_v-semigroups based on (FS)

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الملخص

الهدف من هذه الورقة هو تعريف و تقديم صيغة المثالي الثنائي عالي الضبابية في شبه الزمر عالية الضبابية كتعميم لمفهوم المثالي عالي الضبابية بواسطة استخدام مفهوم المجموعات الشاملة الضبابية للعالم الرياضي"دافاز" ومفهوم المثالي الضبابي في نصف الزمر الموضح في المرجع [13] . اخيراً, نوضح كيف نحصل على العلاقة بين الحلاقات الجزئية عالية الضبابية (عالية المثالية) المعتمدة على الفضاء الضبابي الموضح في المرجع [2] و هذه الدراسة المتمثلة في المثاليات الثنائية عالية الضبابية في شبه الزمر عالية الضبابية و المعتمدة على نظرية المجموعات الضبابية .

Abstract

The aim of this paper is to define and introduce a formulation of a fuzzy hyper-bi-ideal (H_v-Bi-ideal) in fuzzy H_v-semigroup as a generalization of fuzzy hyperideal using a classical fuzzy universal set by Davvaz's approach and a fuzzy ideal in semigroup in [13]. Finally, a relationship between the fuzzy subhyperring (hyperideal) based on the concept of the fuzzy space in [2] and this study about fuzzy H_v-bi-ideal in fuzzyH_v-semigroup using a fuzzy set theory is obtained.

Keywords H_v-ideals, fuzzy universal sets, fuzzy hyperoperation, fuzzy hypergroups, fuzzy semihypergroup, fuzzy ideals, fuzzy hyperideals, fuzzy H_v-ideals, fuzzy space.

1. Introduction and basic concepts:

In [21], Zadeh introduced the concept of fuzzy sets, such that membership degree of an object to a set may take on any value in [0, 1]. He defined concept of the fuzzy theory as a generalization of classical theory in a way that membership degree of an object to a set is not restricted to the integers 0 and 1, but belong [0, 1]. Some mathematicians use the concept of fuzzy subset A_x instead of $\mu_A(x)$. A fuzzy subset A_x is written as $A = \{\langle x, \mu_A(x)\rangle : x \in X\}$. Rosenfeld [20] introduced concept of the fuzzy subgroup A of the given group using the binary operation defined over the group, and he introduced the notation of a fuzzy set defined on a set $X \neq \emptyset$. The concept of hyperstructure theory of the hypergroups introduced by Marty [19], as a new mathematical structure which represents a natural extension to the classical algebraic structure. In classical algebraic structure, the composition of two elements A, B is an element X, but in an algebraic hyperstructure, the composition of two elements A, B is a set $P_*(X)$. The notation of fuzzy algebra defined by Liu [15],[16]. The problem in fuzzy case is how to pick out the generalization from available approaches. In [5],[6],[9],[15],[16],[17],[18] some mathematics have been written on fuzzy theory, and the study continued to fuzzy hyperstructure by Davvaz et al. [3],[4], and Davvaz et al. [7],[8]. The difficulty lies in fuzzy theory are how to carry out the classical concepts to the fuzzy algebra. Davvaz [4] introduced the notions of fuzzy H_v-group as generalization of fuzzy subgroup based on the concept of fuzzy set, he introduced the notation of the fuzzy H_v-subgroup of the given ordinary H_v-group and applied the concept of the fuzzy hyperstructures as generalization of Rosenfeld's definition. Some papers criticized the concept of the fuzzy set, Davvaz introduced a new approach of fuzzy H_v-ideal which is based on the notation of fuzzy hyperalgebra based on Rosenfeld"s definition [20]. The introduction of fuzzy hyperstructures has started with the study of the notations of fuzzy hyperalgebra based on classical fuzzy set by Davvaz et al [3],[4]. The relationship between fuzzy hyperalgebra based on (FS) of Dib [11] with fuzzy H_v-algebra based on fuzzy set was established by Davvaz et al. [7],[8]. Dib et al. [11],[12],[13], introduced some properties of fuzzy theory by a new approach of fuzzy spaces, and established the relation between the introduced fuzzy algebra and classical ones. We recall some symbols and concepts which will be used in this paper:

X: for a non-empty set,

I: for the closed interval [0,1] of real numbers,

L: the Lattice $L = I \times I$ with partial order $(m_1, m_2), (n_1, n_2) \in L$;

(1) $(n_1, n_2) \geq (m_1, m_2)$ if $n_1 \geq m_1$, $n_2 \geq m_2$, whenever $n_1 \neq n_2 \neq 0$.

(2) $(n_1, n_2) = (0,0)$ whenever $n_1 = 0$ or $n_2 = 0$.

Then, the Lattice L is a distributive, but it's not complemented. The notation of the *fuzzy space*(FS) $(H, I) = \{(h, I) : h \in H\}$ defined by Dib [11], he introduced some concepts depended on (FS), for example the concept of the fuzzy subspace U of the fuzzy space (H, I), and the notation of the fuzzy element (h, I) in (FS). In [1] Abdulmula introduced the notions of intuitionistic fuzzy hyperalgebra, from [8] we have the following definition, *a fuzzy universal set* $H^{\times I} = \{\{h\} \times I : h \in H\}$, such that $\{h\} \times I = \{(h, u) : u \in I\}$, and $\{h\} \times I \in H^{\times I}$ is called a fuzzy element. A fuzzy subspace $U = \{(h, u_h) : h \in U_\circ\} \subseteq H^{\times I}$ for $\{U_\circ \subset H; u_h \in I\}$. The relationship between the introduced fuzzy hyperideal and the fuzzy bi-hyperideal in fuzzy semihypergroup based on fuzzy space (FS) is establish in this paper.

1.1 Hyperrings and fuzzy hyperrings

Davvaz et al.[3] [7],[8],[10], give us a new approach of hyperring, fuzzy hypergroup and fuzzy hyperring by using the notation of fuzzy hyperalgebra based on Rosenfeld"s approach [20], if $X \neq \emptyset, P^*(X) = \{A_i \subseteq X, i \in I\}$ then a function (map) $\diamond : X \times X \to P^*(X)$ is called a hyperoperation on X, $(X, \diamond)$ is called a hypergroupoid X_v. If $U, V \subseteq X \neq \emptyset, U \neq V$, we have

$$U \diamond V = \bigcup_{u \in U, v \in V} u \diamond v, x \diamond U = \bigcup_{u \in U} \{x\} \diamond U \quad and \quad U \diamond \{x\} = \{x\} \diamond U.$$

If $\{(x\diamond y)\diamond z = x\diamond(y\diamond z);\ x,y,z \in X\}$, then a hypergroupoid $(X,\diamond)$ is called a semihypergroup, $\bigcup_{u\in x\diamond y} u\diamond z = \bigcup_{v\in y\diamond z} x\diamond v$ We say that the concept of the semihypergroup $(H,\diamond)$ is an ordinary H_v-group if

$\{x \diamond H = H \diamond x = H, x \in H\}$ (reproduction axiom). The triple (X, ◇, •) is called a H_v-ring if

1. $(X,\diamond)$ is a H_v-group,

2. $(X,\bullet)$ is a H_v-semigroup,

3. the H_v-operation • is distributive over the H_v-operation ◇ .

The concepts of fuzzy subhypergroup, fuzzy hyperideal and fuzzy subhypermodule are introduced by Davvaz et al. [3],[4], using Rosenfelds approach [20]. He defined a fuzzy subhypergroup, fuzzy hyperideal and fuzzy hypersubmodule by a given ordinary hypergroup, if X be an ordinary H_v-ring, $U \subseteq X$, then U is called a fuzzy H_v-ideal of X if:

1. $minU(a), U(b) \leq inf_{\psi\in a+b}\{U(\psi)\}$ for all $a, b \in X$,

2. for all $a, c \in X$ there exists $b \in X$ such that $a \in c + b$ and $minU(a), U(c) \leq U(b)$,

3. for all $a, c \in X$ there exists $z \in X$ such that $a \in c \cdot z$ and $minU(a), U(c) \leq U(z)$,

4. $U(b) \leq inf_{\psi\in a\cdot b}U(\psi)$ (respectively $U(a) \leq inf_{\psi\in a\cdot b}U(\psi)$ for all $a, b \in X$.

The notation of fuzzy hypergroup and fuzzy hyperring, using the concept of the fuzzy universal set is investigated by [7],[9], if $X^{\times I} \neq \emptyset$, we have $\langle X^{\times I}, \dot{F}\rangle$is a fuzzy universal set $X^{\times I}$ with a fuzzy hyperoperation $\dot{F}$, then $\langle X^{\times I}, \dot{F}\rangle$ is called a fuzzy hypergroupoid. But if we have two fuzzy hyperoperations over $X^{\times I}$, then the triple $\langle X^{\times I}, \dot{F}, \ddot{F}\rangle$ is a fuzzy H_v-ring if the following hold;

(1) $\langle X^{\times I}, \dot{F}\rangle$ is a fuzzy H_v-group),

(2)) $\langle X^{\times I}, \ddot{F}\rangle$ is a fuzzy H_v-subgroup),

(3) $\ddot{F}$ is distributive over $\dot{F}$.

Theorem 1.1 [14] *For every fuzzy X_v-structure $\langle X^{\times I}, \tilde{F}\rangle$ there is an associated classical X_v-structure (X, F) which is isomorphic to the fuzzy hyperstructure $\langle X^{\times I}, \tilde{F}\rangle$ by the correspondence $x \times \{I\} \leftrightarrow x\ ;\ x \in X$.*

Fuzzy Bi-hyperideal In Fuzzy H_v-semigroup

We define and establish some properties and concepts of the fuzzy hyperalgebra, such as fuzzy bi-hyperideals in fuzzy semihypergroups, using Davvaz's approach [7], and the relationship between these concepts and classical ones is obtained. The concept of the fuzzy H_v-ideal is defined by [2], if $\langle U, \dot{F}\rangle$is a fuzzy H_v-subsemigroup of the fuzzy H_v-

semigroup$\langle (X,I),\dot{F}\rangle$, then$\langle U,\dot{F}\rangle$ is called fuzzy left (right) H_v-ideal where $\dot{F} = (F,f_{xy})$, if the following axioms hold:

$$(u\ I)\dot{F}(v,I_v) = (\{c\},I_c) \subseteq P^*(U) \qquad \Big((v,I_v)\dot{F}(u,I) = (\{c\},I_c) \subseteq P^*(U)\Big).$$

for each $(u,I) \in (X,I),(v,I_v) \in U$.

Definition 2.1 if $(U,\dot{F})$ is a fuzzy H_v-subsemigroup of the fuzzy H_v-semigroup $\langle R^{\times I},\dot{F}\rangle$, then $(U,\dot{F})$ is called fuzzy H_v-bi-ideal where $\dot{F} = (F,f_{xy})$, if the following axioms hold:

$$((\{v\}\times I_v)\dot{F}(\{u\}\times I))\dot{F}\{c\}\times I_c = \{v\}\times I_v\dot{F}(\{u\}\times I\dot{F}\{c\}\times I_c) = \{d\}\times I_d \subseteq P^*(U)$$

, for every $\{v\}\times I_v,\{c\}\times I_c \in U$ and $\{u\}\times I \in R^{\times I}$.

Where, if $\dot{F} = \hat{F}$ be an ordinary fuzzy operation, then $\langle R^{\times[0,1]},\dot{F}\rangle = \langle R^{\times[0,1]},\hat{F}\rangle$ by Dib.

Example 2.3 Let $R = \{\pm 1,\pm i\}$. And if $\dot{F} = (\hat{F},f_{xy})$ is the fuzzy H_v-operation on $R^{\times I}$, we have

$$\hat{F}(u,v) = \begin{cases} \{u.v\} & if\ u,v \in \Re \\ real\ number & if u\ or\ v \in \Re \end{cases},$$

$$f_{11}(m,n) = \begin{cases} \frac{mn}{\sqrt{\omega}} & if\ mn < \omega \\ 1+\frac{mn-1}{1+\sqrt{\omega}} & if\ mn \geq \omega \end{cases},$$

$$f_{-11}(m,n) = f_{1-1}(m,n) = \begin{cases} \frac{mn}{\sqrt{\omega}} & if\ mn < \sqrt{\omega\psi} \\ 1+\frac{1-\sqrt{\psi}}{1-\sqrt{\omega\psi}}(mn-1) & if mn \geq \sqrt{\omega\psi} \end{cases},$$

$$f_{-1-1}(m,n) == \begin{cases} \frac{\sqrt{\omega}}{\psi}mn & if\ mn \leq \psi \\ 1+\frac{1-\sqrt{\omega}}{1-\psi}(mn-1) & if\ mn > \psi \end{cases},$$

the other membership functions are as m.n, where $\omega,\psi \in (0,1)$. Then we have$\langle R^{\times[0,1]},\dot{F}\rangle$ is a fuzzy H_v-semigroup. Also the fuzzy subuniversal set $W = \{(-1,[0,\sqrt{\psi}]),(1,[0,\sqrt{\omega}])\}$ with the fuzzy binary H_v-operation $\dot{F}$ define a fuzzy H_v-subsemigroup of $\langle R^{\times[0,1]},\dot{F}\rangle$. Now, we notice that for all,

$$\big(-1,[0,\sqrt{\psi}]\big),\big(1,[0,\sqrt{\omega}]\big) \in W\ and\ (1,[0,1]) \in R^{\times[0,1]},$$

$$\left((1,[0,\sqrt{\omega}])\dot{F}(1,[0,1])\dot{F}\right)\dot{F}\left(-1,[0,\sqrt{\psi}]\right) =$$

$$(\left(\{-1\},\left[0,1+\frac{1-\sqrt{\psi}}{1-\sqrt{\omega\psi}}\left(\frac{\sqrt{\omega\psi}+\psi}{1+\sqrt{\omega}}-1\right)\right]\right)) \nsubseteq P^*(W) \text{ , and}$$

$$(1,[0,1])\dot{F}\left((1,[0,\sqrt{\omega}])\dot{F}\left(-1,[0,\sqrt{\psi}]\right)\right) =$$

$$\left(\left(\{-1\},\left[0,1+\frac{1-\sqrt{\psi}}{1-\sqrt{\omega\psi}}\left(\frac{1-\sqrt{\psi}}{1-\sqrt{\omega\psi}}-1\right)\right]\right)\right) \nsubseteq P^*(W).$$

Then in this case, a fuzzy H_v-subsemigroup $(W,\dot{F})$ of the fuzzy semigroup $\langle R^{\times[0,1]},\dot{F}\rangle$ is not fuzzy H_v-bi-ideal, that depended on the definition of fuzzy hyperoperation $\dot{F}$. On the other hand, if the comembership functions have the following forms: $\underline{f_{xy}}\ (m,n) = m \wedge n$ Then a fuzzy subsemihypergroup $(W,\dot{F})$ of the fuzzy H_v-semigroup $\langle R^{\times[0,1]},\dot{F}\rangle$ is fuzzy H_v-bi-ideal, i.e.

$$((i,[0,1])\ \dot{F}\ (1,[0,\sqrt{\omega}]))\dot{F}\left(-1,[0,\sqrt{\psi}]\right) = \left(-1,[0,\sqrt{\psi}]\right) \subseteq P^*(W) \text{ , and}$$

$$\left((1,[0,1])\dot{F}\left(1,[0,\sqrt{\omega}]\right)\right)\dot{F}\left(-1,[0,\sqrt{\psi}]\right) = \left(-1,[0,\sqrt{\psi}]\right) \subseteq P^*(W)$$

for all $\left(1,[0,\sqrt{\omega}]\right),\left(-1,[0,\sqrt{\psi}]\right) \in W \quad and \quad (i,[0,1]) \in R^{\times[0,1]}$.

Theorem 2.4 *If* $\langle W,\dot{F}\rangle$ *is a fuzzy* H_v*-subsemigroup of the fuzzy* H_v*-semigroup* $\langle R^{\times I},\dot{F}\rangle$, *then* $(W,\dot{F})$ *is a fuzzy* H_v*-bi-ideal* iff

(1) $(W_\circ,\hat{F})$ *is an ordinary bi-hyperideal of the semihypergroup* $(R,\hat{F})$.
(2) *We have*

$I_{v\hat{F}u\hat{F}c} = f_{(v\hat{F}u)\hat{F}c}(f_{vu}(I_v \times I) \times I_c) = f_{v\hat{F}(u\hat{F}c)}\left(I_v \times f_{uv}([0,1] \times I_c)\right) = I_d$, *for all* $v, c \in W_\circ$ *and* $u \in R$.

Proof 1 Let $(W,\dot{F})$ is a fuzzy bi-hyperideal of the fuzzy semihypergroup $\langle R^{\times I},\dot{F}\rangle$ Then for all $\{v\}\times I_v, \{c\}\times I_c \in W$ *and* $\{u\}\times[0,1] \in R^{\times I}$,
we have

$$\left[\{v\}\times I_v\dot{F}\{u\}\times I\right]\dot{F}\{c\}\times I_c = \{v\}\times I_v\dot{F}\left[\{u\}\times I\dot{F}\{c\}\times I_c\right] \subseteq P^*(W).$$

$$\Leftrightarrow \left[(v\hat{F}u), f_{uv}(I_v\times I)\right]\hat{F}\{c\}\times I_c = \{v\}\times I_v\hat{F}\left[(u\hat{F}v), f_{uv}(I\times I_c)\right] \subseteq P^*(W)$$

$$\Leftrightarrow \left[(v\hat{F}u)\hat{F}c, f_{(v\hat{F}u)\hat{F}c}(\{v\}\times I_v\times I_c)\right]$$

$$= \left[v\hat{F}(u\hat{F}c), f_{v\hat{F}(u\hat{F}c)}\left(I_v\times f_{uc}(I\times I_c)\right)\right] \equiv \{\{d\}\times I_d\} \subseteq P^*(W)$$

$$\Leftrightarrow (i) \quad (v\hat{F}u)\hat{F}c = v\hat{F}\left(u\hat{F}c\right) = \{d\} \subseteq P^*(W_\circ), v, c \in W_\circ, \qquad u \in R,$$

$$(ii) \quad f_{(v\hat{F}u)\hat{F}c}(f_{vu}\{v\}\times I_v\times I_c) = f_{v\hat{F}(u\hat{F}c)}\left(I_v\times f_{uc}(I\times I_c)\right) = I_d.$$

$\Leftrightarrow$ (i) $(W_\circ,\hat{F})$ is a H_v-bi-ideal of H_v-semigroup $(R,\hat{F})$,

$(ii)\ I_{v\hat{F}u\hat{F}c} = f_{(v\hat{F}u)\hat{F}c}(f_{vu}(I_v \times I) \times I_c) = f_{v\hat{F}(u\hat{F}c)}\big(I_v \times f_{uc}(I \times I_c)\big).$

Remark 1 1. If $\dot{F} = \hat{F}$ is a map as fuzzy operation; we have

(i) the concept of the fuzzy semigroup $(H^{\times I}, \hat{F})$ is a fuzzy H_v-semigroup $\langle R^{\times I}, \dot{F}\rangle$,

(ii) the concept of the ordinary H_v-semigroup $(R, \hat{F})$ is a classical semigroup $(H, \hat{F})$,

2. if fuzzy H_v-semigroup $\langle R^{\times I}, \dot{F}\rangle$ is fuzzy ring $(H^{\times I}, \dot{F}, \dot{G})$, where

$\dot{F} = (F, f_{xy}), \dot{G} = (G, g_{xy})$, are two operations on (FS) $H^{\times I}$, then the fuzzy H_v-subsemigroup $\langle U, \dot{F}\rangle$is fuzzy subring $(U, \dot{F}, \dot{G})$ and the ordinary H_v-semigroup (R, F) is an ordinary ring (H, F, G).

Now we introduce the relation between a fuzzy subhyperring(hyperideal) and fuzzy H_v-(Bi-ideal) using (FS) as following;

Remark 2 *The concept of the fuzzy bi-hyperideal is a fuzzy left (right) H_v-ideal in the ordinary (classical) fuzzy theory, but by using (FS) there are left (right) H_v-ideals which are not a fuzzy bi-hyperideals as in the following:*

Example 2.4 Let $R = \{0, 1, 2\}$ and (R, F) is a semihypergroup defined by Table 2.1.

Table 2.1 A fuzzy hyperideal is not a fuzzy bi – hyperideal

F	0	1	2
0	{0}	{1}	{0}
1	{0}	{1}	{0}
2	{0}	{1}	{2}

If $W = \{(0, [0, \frac{1}{2}]), (1, [0, \frac{1}{2}])\}$, then:

(1) In the classical fuzzy theory, we introduce that $W_\circ = \{0, 1\} \subseteq H$ is a hyperideal and a bi-hyperideal of $(R, \hat{F})$.

(2) Consider $\langle R^{\times I}, \dot{F}\rangle$ is a fuzzy semihypergroup, where $\dot{F} = (\hat{F}, f_{xy})$, and

$$f_{xy}(r, s) = \begin{cases} r \wedge s & x = y \\ r \cdot s & otherwise \end{cases},$$

then we have the following:

(i) for $W = \{(0, [0, \frac{1}{2}]), (1, [0, \frac{1}{2}])\}$, we have $f_{00}(I_0 \times I_0) = f_{00}\big([0, \frac{1}{2}] \times [0, \frac{1}{2}]\big) = [0, f_{00}(\frac{1}{2}, \frac{1}{2})] = [0, \frac{1}{2}] = I_0$. and hence $(W, \dot{F})$ is a fuzzy H_v-subsemigroup of $\langle R^{\times I}, \dot{F}\rangle$.

(ii) we have, $f_{x0}(I_x \times I_0) = f_{x0}\left([0,1] \times \left[0, \frac{1}{2}\right]\right) = \left[0, \frac{1}{2}\right] = I_{0F0} = I_0$, then, from (1), we see that $(W, \dot{F})$ is a fuzzy hyperideal

(iii) now, we introduce that W is not a fuzzy bi-hyperideal, for

$$I_0 = I_{1\hat{F}0\hat{F}0} = f_{(1\hat{F}0)0}(f_{10}(I_x \times I_0) \times I_0) = f_{00}\left([0, f_{10}(1,\tfrac{1}{2})] \times I_0\right) = f_{00}([0,1]) \times [0,\tfrac{1}{2}] = \left[0, f_{00}\left(1,\tfrac{1}{2}\right)\right] = \left[0,\tfrac{1}{2}\right] = I_0.$$

However

$$I_{(1F0)F0} \neq I_{1F(0F0)} = f_{1F(0F0)}\left(I_a \times f_{00}(I_0 \times I_0)\right) = f_{10}\left([0,1] \times f_{00}\left(\left[0, f_{00}\left(\tfrac{1}{2},\tfrac{1}{2}\right)\right]\right)\right) = f_{10}(I_x \times I_0) = \left[0, f_{10}\left(1,\tfrac{1}{2}\right)\right] = [0,1] \neq I_{10}$$

That is, $I_a = I_{aFbFa} = f_{a(bFa)}\left(I_a \times f_{ba}(I \times I_a)\right) \neq f_{(aFb)a}(f_{ab}(I_a \times I) \times I_a)$.

Then $(W, \dot{F})$ is a fuzzy left H_v-ideal of $\langle R^{\times I}, \dot{F}\rangle$, but it is not a fuzzy H_v-bi-ideal of $\langle R^{\times I}, \dot{F}\rangle$.

According to the above properties we introduce the relation between the fuzzy hyperideals and the fuzzy H_v-bi-ideal. Now, we give the necessary condition for the fuzzy (left, right) hyperideal to be a fuzzy H_v-bi-ideals.

Theorem 2.5 *Every fuzzy left hyperideal W of the fuzzy semihypergroup $\langle R^{\times I}, \dot{F}\rangle$ is a fuzzy bi-hyperideal if*

(1) $f_{yc}(I_y \times I_c) = I_{y\hat{F}c} = I_{\{d\}}$,
(2) $I_{y\hat{F}x\hat{F}c} = f_{(y\hat{F}x)c}\left(f_{yx}(I_y \times I) \times I_c\right), such\ that\ \ y, c, w\ \in W_\circ\ \ and\ x \in R.$

Proof 2 If $W \subseteq \langle R^{\times I}, \dot{F}\rangle$ is a fuzzy left H_v-ideal. By the theorem 2.3, we get

(i) $(W_\circ, \dot{F})$ is the H_v-ideal of the classical semihypergroup which implies $(W_\circ, \hat{F})$ is a H_v-bi-ideal of $(R, \hat{F})$,
(ii) we have $f_{xy}(I \times I_c) = I_{xFz},\ \ \forall\, x\ \in\ R\ , c\ \in\ W_\circ$,

since $x\hat{F}c\ \in\ W_\circ\ ,\ y\hat{F}(x\hat{F}\ c) \in\ W_\circ,\ \ \forall x \in R,\ y, c \in W_\circ$, then by condition (1), we have

$$f_{y(x\hat{F}c)}(I_y \times I_{x\hat{F}c}) = I_{y\hat{F}x\hat{F}c} = I_{\{w\}},$$

from (i) and (ii), and the condition (2), we get

$$I_{y\hat{F}x\hat{F}c} = f_{y(x\hat{F}c)}(I_y \times I_{x\hat{F}c}) = \left(I_y \times f_{xc}(I \times I_c)\right) = f_{(y\hat{F}x)c}(f_{yx}(I_y \times I) \times I_c).$$

Then W is a fuzzy H_v-bi-ideal of $\langle R^{\times I}, \dot{F}\rangle$.

Corollary 2.6 *Every fuzzy H_v-ideal $\langle W, \dot{F}\rangle$ of the fuzzy H_v-semigroup $\langle R^{\times I}, \dot{F}\rangle$ is a fuzzy bi-hyperideal if $f_{yz}(I_y \times I_z) = I_{y\hat{F}z}$; $\forall y, z \in W_\circ$.*

2.1. Fuzzy Bi-hyperideal Using The Fuzzy Universal Subset

If $S \neq \emptyset, S \subseteq R$, and let $R(S) \subseteq R^{\times I}$, where $R(S)$ is induced, then by the fuzzy set S, then we get the following;

Theorem 2.7 *Let $\langle R^{\times I}, \dot{F}\rangle$ is a fuzzy semihypergroup, and $\boldsymbol{R}(S)$ is a fuzzy subsemihypergroup of h $\langle R^{\times I}, \dot{F}\rangle$, then $\boldsymbol{R}(S)$ is a fuzzy bi-hyperideal iff*

(1) $(S_\circ, \hat{F})$ *is a H_v-bi-ideal of the classical semihypergroup* $(R, \hat{F})$,

(2) *we have,* $S(yF\, xF\, z) = f_{(yF\, x)z}\big(f_{yx}(Sy, 1), Sz\big) = f_{y(xF\, z)}\big(Sy, f_{xz}(1, Sz)\big), \forall y, z \in S_\circ, x \in R.$

Remark 3

If we have a function (map) as fuzzy operation $\dot{F} = \hat{F}$, then the fuzzy H_v-semigroup $\langle R^{\times I}, \hat{F}\rangle$ is fuzzy semigroup $\left(X^{\times I}, \hat{F}\right)$, and the ordinary semigroup

(1) $(R, \hat{F})$ is an ordinary semigroup $\left(X, \hat{F}\right)$ and $\left(\boldsymbol{R}(S), \dot{F}\right)$ is a fuzzy subsemigroup of the fuzzy semigroup $\left(X^{\times I}, \hat{F}\right)$.

(2) Let the fuzzy semihypergroup $\langle R^{\times I}, \dot{F}\rangle$ is a fuzzy ring $(X^{\times I}, \dot{F}, \dot{G})$, there are two operations $\dot{F} = (\hat{F}, f_{xy})$, $\dot{G} = (\hat{G}, g_{xy})$, on the fuzzy space $X^{\times I}$, then the fuzzy subsemihypergroup $(R(S), \tilde{F})$ is fuzzy subring $(R(S), \tilde{F}, \dot{G})$ and the ordinary semihypergroup $(R, \hat{F})$ is an ordinary ring $(X, \hat{F}, \hat{G})$ by Dib.

Now we introduce the interesting relationship between the fuzzy H_v-bi-ideal and an ordinary (classical) fuzzy H_v-bi-ideals as the following theorem;

Theorem 2.8 *If we have a uniform fuzzy semihypergroup $\langle R^{\times I}, \dot{F}\rangle$, $\dot{F} = \left(\hat{F}, f_{xy}\right) = \left(\hat{F}, \wedge\right)$, $\wedge: I \times I \to I$ where $\wedge$ is a minimum function, then every fuzzy set $S \subseteq R$ which induces a fuzzy bi-hyperideal $\langle R(S), \dot{F}\rangle$ of $\langle R^{\times I}, \dot{F}\rangle$is an ordinary fuzzy bi-hyperideal of the semigroup* $(R, \hat{F})$.

Proof 3 Let $\langle R^{\times I}, \dot{F}\rangle$ is an uniform fuzzy H_v-semigroup, where $\dot{F} = (\hat{F}, \wedge)$, and let $S \subseteq R$, $(R(S), \dot{F})$ is a fuzzy H_v-bi-ideal of $\langle R^{\times I}, \dot{F}\rangle$, then using Theorem 2.7, we have

(i) $(S_\circ, \hat{F})$ is a H_v-bi-ideal of the ordinary H_v-semigroup $(R, \hat{F})$,

(ii) for all $w, z \in S_\circ$ and $x \in R$, we have

$$S\big(w\hat{F}x\hat{F}z\big) = \big(S(w)\hat{F}S(x)\big)\hat{F}S(z) = \big(S(w) \wedge S(x)\big) \wedge S(z) = (S(w) \wedge 1) \wedge S(z) = \big(S(w) \wedge S(z)\big). \quad (*)$$

If $w, z \notin S_\circ$, then $S(w) = S(z) = 0, \forall\ x, w, z \in R$,

$$S(w\hat{F}\, x\hat{F}\, z) \geq (S(w) \wedge S(z)),$$

then S satisfies Davvaz's definition in (2006) of intuitionistic bi-hyperideal in semihypergroup, hence S is a classical fuzzy H_v-bi-ideal of $(R, \hat{F})$.

Remark 4 (1) If the concept of the uniform fuzzy H_v-semigroup $\langle R^{\times I}, \dot{F}\rangle$ is a fuzzy semigroup $(X^{\times I}, \hat{F})$, then $(R(S), \dot{F})$ is an ordinary fuzzy bi-ideal of the classical fuzzy semigroup $(X^{\times I}, \hat{F})$.

(2) If the fuzzy semihypergroup $\langle R^{\times I}, \tilde{F}\rangle$ is an uniform fuzzy ring $(X^{\times I}, \tilde{F}, \tilde{G})$, then the fuzzy H_v-bi-ideal $\langle R(S), \dot{F}\rangle$ is fuzzy subring $(R(S), \dot{F}, \dot{G})$ and the ordinary H_v-semigroup $(R, \hat{F})$ is a classical ring $(X, \hat{F}, \hat{G})$.

(3) generally, the converse of the last theorem is not true, as we see the following example:

Example 2.9 Let $\langle R^{\times I}, \dot{F}\rangle$ is an uniform fuzzy semihypergroup, where $(R, \hat{F})$ is the semihypergroup as in Table 2.2, and $f(r,s) = r \wedge s$, where $R = \{e, a, b\}$.

Table 2.2 A classical fuzzy $Hv - Bi -$ ideal

$\dot{F}$	e	a	b
e	$\{e\}$	$\{a\}$	$\{b\}$
a	$\{a\}$	$\{a\}$	$\{b\}$
b	$\{a\}$	$\{a\}$	$\{b\}$

Let S is a fuzzy set, $S \subseteq R$, where $S(e) = \frac{1}{3}, S(a) = \frac{1}{8}, S(b) = 0$, then we have:

(i) S is an ordinary fuzzy H_v-bi-ideal of $(R, \hat{F})$,
(ii) the suppose $S_\circ = \{\{e, a\}; S_\circ \subseteq S\}$ is a H_v-bi-ideal of $(R, \hat{F})$,
(iii) the support $S_\circ \subseteq S$ is a H_v-bi-ideal of $(R, \hat{F})$ and induced the fuzzy subspace $R(S) = \{(e, [0, \frac{1}{3}]), (a, [0, \frac{1}{8}])\}$,

we introduce that S is not a fuzzy H_v-bi-ideal in our sense, because for all

$a, e \in S_\circ, b \in R$, then by $(*)$ in Theorem 2.8, we get

$$S(e\hat{F}\, a\hat{F}\, b) = S(b) = 0 \neq S(e) \wedge S(a) = \frac{1}{8}.$$

Corollary 2.10 *Any ordinary fuzzy H_v-bi-ideals S of the classical hypergroup $\langle X, \hat{F}\rangle$ induces fuzzy H_v-bi-ideal relative to some fuzzy H_v-bi-ideals $\langle X^{\times I}, \dot{F}\rangle$.*

3 Conclusion

In this paper, we have generalized the notation initiated by [13] about fuzzy bi-ideal in fuzzy semigroup to the context of fuzzy H_v-(bi-ideal) in fuzzy H_v-semigroups based on (FS). We get a relationship between fuzzy subhyperring (hyperideal) based on fuzzy space [2] and fuzzy H_v-bi-ideal in fuzzy H_v-semigroup using a fuzzy universal set theory is obtained. We get a relationship between the induced fuzzy H_v-bi-ideal (fuzzy H_v-bi-ideal) in fuzzy H_v-semigroup using a fuzzy space and the concept of the fuzzy (H_v-bi-ideal) based on fuzzy universal sets by Davvaz's approach [3] is established.

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